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Parameters of the Cubic Equation Solving

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Abstract

Cardano investigated del Ferro's papers and found that he had discovered the solution to the cubic before Tartaglia. Anxious to publish the solutions to the cubic, and not wanting to betray Tartaglia, Cardano used the fact that del Ferro had discovered the solution first to evade his promise of secrecy. Although Cardano expanded on his Islamic predecessors by including the possibility of negative solutions, he was still not able to find all the possible solutions to cubic equations. Rafael Bombelli continued to expand on, and refine, the work of Cardano. Bombelli wrote a book that contained a logical progression from linear to quadratic to cubic equations. He also included the idea that it seemed possible to take the square root of a negative number, something that is encountered when using Cardano's formula for some cubic equations. He used notation that was a stepping stone toward the notation currently used in algebra. For example, Bombelli began writing R.Sq. to represent the square root. Bombelli also noted that it was possible to take the cube root of numbers that were negatives. He noted that this required a, different set of rules for these new numbers, which he called "plus of plus" and "minus of minus" (Katz, 1998). In modern terms, Bombelli had begun working with imaginary numbers.

Keywords: Equation, Evolution, Cubic

Introduction

Mathematician Fra Luca Pacioli noted that there was not yet a solution to the general cubic equation in his book Summa de Arithmetics in the year 1494 (Dunham, 1990). Specifically, Pacioli was of the opinion that finding a solution to the general cubic was as likely as squaring the r circle (Dunham, 1990)8. However, many mathematicians were working on this problem during the fifteenth and sixteenth centuries. Sciopine del Ferro took up the challenge to find the solution to the general cubic while teaching at the University of Bologna between 1500 and 1515 (Katz, 1998). In fact, del Ferro did find a method for solving the cubic, but not to the general form. The general form of a cubic would now be written as 8 Recall that squaring a circle was once considered an extremely difficult task, and was later proven to be impossible. (For more information, see: Dunham, W. (1990).[8] Journey through Genius. New York, NY: Penguin Books.)

$ox^3 + Z>x^2 + ex + d = 0$.

The solution method del Ferro found solved depressed cubics, those without a square term:

$ax^3 + ex + d = 0$.

Curiously, his method of finding a solution was not publicized, but rather, was kept a secret. This secrecy was a function of the academic attitude at the time. The current trend in academia is for professors to publish new results as quickly and often as possible. However, in the sixteenth century, university professors were expected to challenge others, and to meet the challenges of others.

Their professorial worth was on the line every time they took up a challenge, as was the security of their jobs. For this reason, del Ferro did not publish his results. Rather, he kept his breakthrough a secret shared with no one but his student Antonio Maria Fior, and his successor Annibale della Nave, whilst on his deathbed. Fior and Nave did not publicize the solution, but word spread that the solution to the cubic was known. Soon another mathematician named Niccolo Fontana best known as "Tartaglia") boasted that he too knew the solution to the cubic.

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Fior publicly challenged Fontana in 1531. Each mathematician provided problems for the other to solve. For example, "A man sells a sapphire for 500 ducats, making a profit of the cube root of his capital. How much is that profit?" This problem could be written algebraically as

$$x^3 + x = 500 .$$

Tartaglia discovered the solution to this cubic problem, while Fior was unable to solve many of Tartaglia's noncubic mathematical questions (Katz, 1998). For this reason, Tartaglia was declared the winner of the mathematical duel. His prize was 30 banquets prepared by Fior, which Tartaglia declined in favor of simply having the honor of being the victor (Katz, 1998). Gerolamo Cardano, a mathematician giving public lectures on mathematics in Milan, heard about Tartaglia's solution to the cubic. After many entreaties, Tartaglia agreed to share his method with Cardano, provided that Cardano would not publish these methods. This is how Tartaglia solved the cubic $x^3+cx = d$.

1When the cube and its things near
2Add to a new number, discrete,
3Determine two new numbers different
76
4 By that one; this feat
5Will be kept as a rule
6Their product always equal, the same,
7To the cube of a third
8Of the number of things named.
9Then, generally speaking,
10The remaining amount
110f the cube roots subtracted
12Will be your desired count.
(Katz, 1998, p359)

Line 1 refers to x3 (the cube) and ex (its things). In Line 2, Tartaglia is referring to creating the term x3+cx = d. Let v and w be the two new numbers in Line 3. Let their difference be represented by v-w-d in this line. Lines 6 and 7 refers to the term v.w.=$\left(\frac{c}{3}\right)^3$ Thus, Lines 9 through 12 say that $\sqrt[3]{v} - \sqrt[3]{w}$ is the solution to the problem. This can be checked by substituting this solution into the original cubic $x^3+cx = d$. This would initially give the expression

$$\left(\sqrt[3]{v}-\sqrt[3]{w}\right)^3 + c\left(\sqrt[3]{v}-\sqrt[3]{w}\right),$$

which should be equal to "d." Expanding this expression leads to

$$v-w-3\sqrt[3]{v}\sqrt[3]{vw}+3\sqrt[3]{w}\sqrt[3]{vw}+c\left(\sqrt[3]{v}-\sqrt[3]{w}\right).$$

Substituting v-w = d and $\left(\frac{c}{3}\right)^3$ gives

$$d-3\sqrt[3]{v}\sqrt[3]{\left(\frac{c}{3}\right)^3}+3\sqrt[3]{w}\sqrt[3]{\left(\frac{c}{3}\right)^3}+c\sqrt[3]{v}-c\sqrt[3]{w}.$$

This can be simplified to
d$-c\sqrt[3]{v}+c\sqrt[3]{w}+c\sqrt[3]{v}-c\sqrt[3]{w}+$
which becomes “d.” Thus, $\sqrt[3]{v}-c\sqrt[3]{w}$ is a solution to the equation $x^3+cx = d$. The question then becomes how to find the values for v and w. This will be discussed in the geometric derivation that follows. Although this method is not the traditional method used to solve cubic equations, it has a stronger appeal in terms of applications to the classroom.

Cardano and his student, Lodovico Ferrari, continued working on solutions to the various forms of the cubic (where a variation of the 4 terms would be missing).

Cardano investigated del Ferro's papers and found that he had discovered the solution to the cubic before Tartaglia. Anxious to publish the solutions to the cubic, and not wanting to betray Tartaglia, Cardano used the fact that del Ferro had discovered the solution first to evade his promise of secrecy. Cardano published Ars Magna, sive de Regulis Algebraicis (The Great Art, On the Rules of Algebra) (Katz, 1998). Cardano's Formula (in modern notation) to solve the cubic $x^3+cx = d$ is

$$x=\sqrt[3]{\frac{d}{2}+\sqrt{\frac{d^2}{4}+\frac{c^3}{27}}}-\sqrt[3]{-\frac{d}{2}+\sqrt{\frac{d^2}{4}+\frac{c^3}{27}}}\ .$$

Before examining Cardano's method, it is interesting to note that both Niccolo Fontana and Gerolamo Cardano both led fascinating lives. Fontana was disfigured as a boy when a soldier slashed his face with a sword. The legend tells that he survived only because a dog licked his wound, causing it to heal (Dunham, 1990). Due to the serious injury to his face, Fontana had a speech impediment. His nickname became Tartaglia (The Stammerer), and he is best known by that nickname today (Dunham, 1990). Gerolamo Cardano was plagued by infirmities throughout his life (Dunham, 1990). He kept track of his many afflictions, and 79 left a detailed accounting of them in his autobiography.

Cardano had a vision of a woman in white in a dream. When he later met a woman that he felt resembled that of his dream, he married her. When his wife died, leaving him with two sons and a daughter, Cardano was left to raise his children alone. He writes that disaster struck in the form of a "wild woman," whom his eldest son Giambattista married (Dunham, 1990). The couple soon produced a male child named Fazio. Unfortunately, the wife boasted that none of the children were Giambattista's. Giambattista prepared a cake laced with arsenic that killed his wife. He was subsequently convicted and beheaded. Cardano raised Fazio as a son, and the relationship thrived. Near the end of his life, Cardano was jailed for heresy against the Church of Italy for several issues, including writing a book titled In Praise of Nero. These two mathematicians of the 16th century led fascinating lives, and their stories serve as a reminder that mathematicians are humans too. Tartaglia and Cardano both played important roles in deriving the solution to cubic equations. Cubics of the form $x^3+cx = d$ are considered "depressed" because the square term is missing. Cubics that begin in the form $x^3+bx^2 + cx = d$ can all be rewritten as depressed cubics. In fact, substituting

$$x = m - \frac{b}{3}$$

(where b is the coefficient of the second degree term) results in a cubic in m with no square term. Substituting this value for x yields

$$\left(m-\frac{a}{3}\right)^3+b\left(m-\frac{a}{3}\right)^2+c\left(m-\frac{a}{3}\right)=d\ .$$

Simplifying leads to

$$m^3-bm^2+\frac{b^2}{3}m-\frac{b^3}{27}+bm^2-\frac{2b^2}{3}m+\frac{b^3}{9}+cm-\frac{cb}{3}=d\ .$$

Finally, by grouping the terms, the equation becomes

$$m^3+(-b+b)m^2+\left(\frac{b^2}{3}-\frac{2b^2}{3}+c\right)m=d-\left(-\frac{cb}{3}\right).$$

The coefficients of the square term become zero, thus creating a cubic with no square term. This is a depressed cubic, for which it is possible to use Cardano's Formula. Hence, it is possible to find a solution to all cubic equations using Cardano's Formula. Keep in mind that all cubic graphs have at least one real root, as they are odd functions. Quantitatively this

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means that the range of the graph will be all real numbers, guaranteeing hence that the graph will have at least one real root. The Intermediate Value Theorem (commonly covered in calculus) guarantees that there exists at least one real root. The Intermediate Value Theorem states that if f(x) is continuous on the interval [a,b], and k lies between f(a) and f(b), then f(x) will have value k for some value of x on the interval [a,b]. Essentially, the function cannot get from one point to the other without crossing horizontal line y = k .

Note that this geometric derivation ties together with Tartaglia's poem to find the solution of a cubic (discussed earlier). In fact, w^3 (the small cube) corresponds to Tartalia's w^3 . In addition, x^3 corresponds to Tartaglia's v^3. Geometrically, this would refer to a cube and a rectangle added together to create "d." Let Figure 1 (blue cube) represent x^3. Then Figure 2 (red cube) is the cube that is missing after the pieces are put together represent m^3 (the piece will not be put together with the rest). Two of Figure 3, the pink rectangular prisms representing ux(u + x), are necessary to complete the picture. Figure 4, the yellow prism represents x^2u. Figure 5, the green rectangular prism represents xu^2. When all the figures are cut out and attached to each other (excluding Net B), they comprise a large cube with dimensions $(x + w)^3 — u^3$. Thus, the three dimensional shape implies that the large cube (which is missing a small cube) is equal to the sum of its parts.

This shape is analogous to the quadratic "gnomon," and might therefore be named a "cubic gnomon."

$(x + w)^3 — w^3 = x^3 + 3x^2w + 3xw^2$.

Factoring produces

$(x + m)^3 - u^3 = x^3 + xu[3(x +u)]$.

Regrouping the terms leads to

$(x + u)^3 — u^3 = x^3 + x[3m(x + w)]$.

This sum represents a cube and six rectangular prisms added together. In other words, this sum is the "d" in the original equation $x^3+cx = d$. Thus, 3u(x + u), the rectangles, is the value "c" in the equation. Consequently, it is possible to write "d" as the original form from which $x^3 + x[3w(x + w)]$ was derived,

$(x + w)^3 - u^3$.

Hence,

$d = (x + u)^3-u^3$ and $c = 3w(x + w)$

This implies that

$$x + u = \frac{c}{3u} .$$

With substitution, it is possible to derive that

$$d = \left(\frac{c}{3u}\right)^3 - u^3 .$$

Cubing each term leads to

$$d = \frac{c^3}{27u^3} - u^3 .$$

Multiplying both sides by u^3 generates

$$d \cdot u^3 = \frac{c^3}{27} - u^6 .$$

Writing an equation in terms of u leads to

$$\frac{c^3}{27} = u^6 + du^3 .$$

In an effort to write a quadratic in u^3 , substitute $y = u^3$ into the previous equation. This leads to

$$y^2 + dy = \frac{c^3}{27}.$$

The Quadratic Formula provides the solution

$$y = -\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}.$$

Notice that the negative solution is not included, as negative solutions were disregarded at the time. From which is follows that

$$u^3 = -\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}.$$

Taking the cube root of both sides leads to

$$u = \sqrt[3]{-\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}}.$$

Recall that originally $d = (x + u)^3 - u^3$. Solving the equation for x gives $x=\sqrt[3]{d + u^3} - u$. Substituting the derived values for "u" and "u^3" provides

$$x = \sqrt[3]{d + \left(-\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}\right)} - \sqrt[3]{-\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}}.$$

This leads to Cardano's solution for x

$$x = \sqrt[3]{\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}} - \sqrt[3]{-\frac{d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}}.$$

Applications to the Classroom

The cube root is analogous to the square root, which is described in Figure 1. The square root refers to the length of a side of a square with a desired area, while the cube root refers to the length of a side of a cube with a desired volume.

Although Cardano expanded on his Islamic predecessors by including the possibility of negative solutions, he was still not able to find all the possible solutions to cubic equations. Rafael Bombelli continued to expand on, and refine, the work of Cardano. Bombelli wrote a book that contained a logical progression from linear to quadratic to cubic equations. He also included the idea that it seemed possible to take the square root of a negative number, something that is encountered when using Cardano's formula for some cubic equations. He used notation that was a stepping stone toward the notation currently used in algebra. For example, Bombelli began writing R.Sq. to represent the square root. Bombelli also noted that it was possible to take the cube root of numbers that were negatives. He noted that this required a, different set of rules for these new numbers, which he called "plus of plus" and "minus of minus" (Katz, 1998). In modern terms, Bombelli had begun working with imaginary numbers.

Bombelli's promotion of the existence of imaginary numbers allowed the use of Cardano's Formula even when the sum beneath the root would be negative.

Problem 1: Use Cardano's Formula to solve a cubic equation.

Use Cardano's Formula to get one real solution of the equation $x^3+63x=316$. Then, use this solution to find
the remaining solutions to the equation.

The Solution:

Cardano's Formula gives

$$x=\sqrt[3]{\frac{d}{2}+\sqrt{\frac{d^2}{4}+\frac{c^3}{27}}}-\sqrt[3]{-\frac{d}{2}+\sqrt{\frac{d^2}{4}+\frac{c^3}{27}}}$$

as the solution to equations of the form

$x^3 + ex = d$.

Thus, substituting the values of c and d into Cardano's formula gives

$$x=\sqrt[3]{\frac{316}{2}+\sqrt{\frac{316^2}{4}+\frac{63^3}{27}}}-\sqrt[3]{-\frac{316}{2}+\sqrt{\frac{316^2}{4}+\frac{63^3}{27}}}\ .$$

Simplifying this equation results in

$$x=\sqrt[3]{343}-\sqrt[3]{27}=7-3=4\ .$$

And so it follows that x-4 is a factor of the original cubic. It is now possible to use long division or synthetic division to determine the quadratic by which x-4 would be multiplied in order to obtain the 1 original cubic. The division of x^3-63x-316 by x-4 results in x^2+4x + 79. And so it follows (by the Quadratic Formula) that the remaining solutions of the original cubic are $-2 \pm 5/\sqrt{3}$.

In some problems, students will find themselves faced with the issue of taking the cube root of a value they are unfamiliar with. Here is an example that would be the first step in solving this type of problem.

Problem 2: An Example Involving the Cube Root of an Imaginary Number.

a) Verify that $(5+ i)^3$ =110 + 74i.

b) Conclude that $\sqrt[3]{110-74i} = 5+i$.

c) Similarly show that $\sqrt[3]{110-74i} = 5-i$.

d) Use Cardano's Formula to find one real solution to x^3-78x=220.

The Solution:

a) Using the binomial theorem, expand

$(5+i)^3=(5)^3+3(5)^2(i)+3(5)(i)^2+(i)^3$.

Simplifying this expression gives the desired result

$(5+ i)^3$ =110 + 74i

b) To show that

$\sqrt[3]{110-74i} = 5-i$

first cube both sides

$110-74i = (5-i)^3$.

Then use the binomial theorem to expand the expression

$(5-i)^3=(5)^3 +3(5)^2(-i)+3(5)^2(-i)+3(5)(-i)^2+(-i)^3$.

Simplifying this expression gives the desired result

$(5-i)^3$ =110-74i.

c) Substitute c=-78 and d=220 into Cardano's Formula to obtain the real solution

$$x=\sqrt[3]{\frac{220}{2}+\sqrt{\frac{220^2}{4}+\frac{(-78)^3}{27}}}-\sqrt[3]{-\frac{220}{2}+\sqrt{\frac{220^2}{4}+\frac{(-78)^3}{27}}}\ .$$

Simplifying the terms under the cube root leaves

$$x=\sqrt[3]{110+\sqrt{-5476}}-\sqrt[3]{-110+\sqrt{-5476}}\ .$$

Rewriting in complex form results in

$$x=\sqrt[3]{110+74i}-\sqrt[3]{-110+74i}\ .$$

Factoring a negative one out of the second cube root gives

$$x = \sqrt[3]{110+74i} + \sqrt[3]{110-74i}\ .$$

Note that $\sqrt[3]{-110+74i} = \sqrt[3]{(-1)(110-74i)} = -\sqrt[3]{110-74i}\ .$

Substituting the known quantities from parts a) and

b) leaves

x = (5 + z)+(5-z).

Hence, one of the solutions to the cubic is

x = 10 .

Problem 3: Large templates for building the geometric representation for the equation $x^3+6x = 20$ are provided. Create the three dimensional figure that represents the equation. Note that the figures are drawn to the scale necessary to create the geometric representation of this equation. Students could actually measure the side length of the appropriate pieces to find the desired solution.

Conclusions

The conclusion of the mathematical problems undertaken in this research work is discussed. The study of the solution STUDY ON THE PARAMETERS RELATED TO THE EVOLUTION OF SOLVING EQUATION OF CUBIC EQUATION on the literature of the research findings in this area are reported. The motivation and the main objectives of this thesis are given. In this section, the essence of the present research work is briefed. Organization of the thesis shows the arrangements of the outcome of the present research work.

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